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INFLUENCE OF HEAT CONDUCTION OF THE WALL ON THE TURBULENT PRANDTL NUMBER IN THE VISCOUS SUBLAYER

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Temperature pulsations in the viscous sublayer and in the heat-conducting wall are analyzed. The analytical dependence of the criterion Pr_t on the parameters Pr and Λ and the coordinate y_+ is determined.

In [1-2] it is shown that in the viscous sublayer of a turbulent boundary stream the characteristics of the wall material affect the magnitude of the temperature pulsations, and a dimensionless criterion is obtained for this effect: $\Lambda = \sqrt{(\rho c_p \lambda)_2/(\rho c_p \lambda)_1}$. The influence of the molecular Prandtl number (or the Schmidt number Sc in the case of mass transfer [3]) on the turbulent transfer in the viscous sublayer was investigated theoretically in [3-5]. The influence of the wall material was partially taken into account in [3-5] by setting up different boundary conditions: of the first kind $[\theta(y=0)=0]$ or of the second kind $[(\partial\theta/\partial y)(y=0)=0]$. This corresponds to $\Lambda=\infty$ and $\Lambda=0$. In the present paper the theory of [3-5] is generalized to arbitrary values of Λ.

We will start from the following equations for the temperature pulsations:

$$\frac{\partial\theta}{\partial t} + v\frac{dT}{dy} = a\frac{\partial^2\theta}{\partial y^2} \quad (y>0), \tag{1}$$

$$\frac{\partial\varphi}{\partial t} = b\frac{\partial^2\varphi}{\partial y^2} \quad (y<0), \tag{2}$$

$$\theta = \varphi \quad (y=0), \tag{3}$$

$$\lambda_1\frac{\partial\theta}{\partial y} = \lambda_2\frac{\partial\varphi}{\partial y} \quad (y=0). \tag{4}$$

Equation (1) describes the temperature pulsations in the viscous sublayer; (2) is the equation of heat propagation in the solid wall; the conditions (3)-(4) express the continuity of the temperature and of the heat flux at the boundary. In (1) we neglected the dependence of v and θ on the coordinates x and z. The applicability of such an approximation can be justified rather rigorously in the case of large Prandtl numbers (see [3-5]), but

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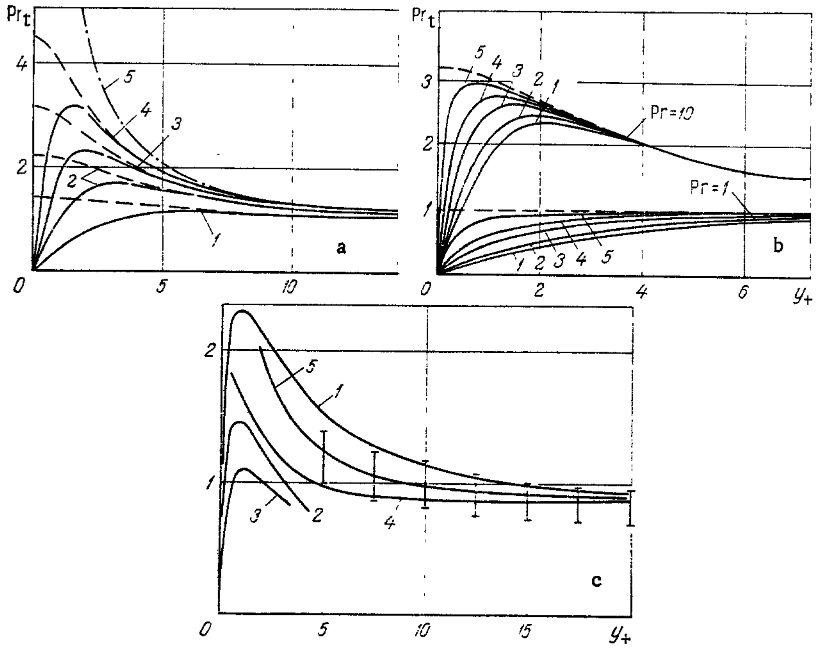

Fig. 1. Turbulent Prandtl number: a) solid curves) $\Lambda = 0$; dashed curves) $\Lambda = \infty$; curves 1-5 correspond to Pr = 2, 5, 10, 20, and ∞, respectively; b) the solid curves 1-5 correspond to $\Lambda = 0$, 0.5, 2, 5, and 20, respectively, while the dashed curves correspond to $\Lambda = \infty$; c) 1, 4) Pr = 8; 2) 10; 3) 5.4; 5) 1000. $\Lambda = 22$.

Eq. (1) can still be used at $Pr \approx 1$ by treating it as a model equation reflecting the main features of the turbulent heat transfer.

To describe the longitudinal velocity pulsations one can use an analogous (model) equation with the usual boundary condition of "attachment" to the wall (see [3]):

$$\frac{\partial u}{\partial t} + v\frac{dU}{dy} = \nu \frac{\partial^2 u}{\partial y^2} \quad (y > 0), \tag{5}$$

$$u = 0 \quad (y = 0). \tag{6}$$

Let us briefly describe the way of solving problem (1)-(4). We write the general solution of Eq. (1) in the form

$$\theta(y, t) = -\int_{-\infty}^{t} dt' \int_{0}^{\infty} dy' \frac{v(y', t')\frac{dT}{dy'}}{\sqrt{4\pi a(t-t')}} \left\{\exp\left[-\frac{(y-y')^2}{4a(t-t')}\right] + A\exp\left[-\frac{(y-y')^2}{4a(t-t')}\right]\right\}, \tag{7}$$

where t is the current time; t' is the time at past moments, since here and later all the random processes are assumed to be statistically steady in time; the lower limit of integration over t' (the start) is taken as $-\infty$.

The solution (7) is constructed from the fundamental solution of the heat-conduction equation and hence it satisfies Eq. (1). The first exponential in the braces gives the particular solution of the inhomogeneous equation (1) while the second assures, as is shown below, that the boundary conditions (3)-(4) are satisfied. In the case of boundary conditions of a general form (explicitly depending on time, for example) the quantity A could depend on the integration variables y' and t', but for the conditions (3)-(4) it is quite sufficient to take A = const.

Taking A = const and y = 0, from (7) we obtain the temperature pulsations at the boundary:

$$\theta_0(t) = -(1 + A)\int_{-\infty}^{t} dt' \int_{0}^{\infty} dy' \frac{v(y', t')\frac{dT}{dy'}}{\sqrt{4\pi a(t-t')}} \exp\left[-\frac{y'^2}{4a(t-t')}\right]. \tag{8}$$

The general solution of Eq. (2) with the condition (3) has the form [6]

$$\varphi(y, t) = \int_{-\infty}^{t} dt'' \frac{|y|\theta_0(t'')}{\sqrt{4\pi b (t-t'')^3}} \exp\left[-\frac{y^2}{4b(t-t'')}\right] \quad (y<0). \tag{9}$$

We substitute (8) into (9) and change the order of integration over t" and t'. After this the integral over t" can be calculated using the theorem on the convolution from the theory of Laplace transforms. As a result,

$$\varphi(y, t) = -(1+A)\int_{-\infty}^{t} dt' \int_{0}^{\infty} dy' \frac{v(y', t')\frac{dT}{dy'}}{\sqrt{4\pi a(t-t')}} \exp\left[-\frac{(|y|\sqrt{a}+y'\sqrt{b})^2}{4ab(t-t')}\right]. \tag{10}$$

Substituting Eqs. (7) and (10) into the boundary condition (4) gives the equation

$$A = (1-\Lambda)/(1+\Lambda).$$

Thus, the solution of problem (1)-(4) is fully determined.

Let us multiply (7) by v(y, t) and average it (over time or over a statistical ensemble). As a result, for the turbulent heat flux we have the expression

$$\langle v\theta \rangle = -\int_0^\infty d\tau \int_0^\infty dy' G(y, y';\ \tau) \langle v(y, t)\, v(y', t-\tau) \rangle \frac{dT}{dy'} = -a_t \frac{dT}{dy}, \tag{11}$$

where the angle brackets denote averaging; G(y, y'; τ) is Green's function of problem (1)-(4) for $y>0$; $\tau = t - t'$. Expression (11) shows that a_t is not simply a function but an integral operator acting on the average temperature gradient. But we will not take this property of the coefficient a_t into account in future, assuming in the first approximation that dT/dy, as a slowly varying function of the coordinates, can be taken outside the integral.

Thus, from Eq. (11) we get a definite expression connecting a_t and the correlation function of the transverse velocity pulsation. An analogous expression can also be obtained for the turbulent viscosity ν_t. In solving this problem one can use the same Green function as in Eq. (7), replacing a by ν and setting A = —1 in it, since the boundary condition (6) is valid for u.

One must determine the form of the velocity correlation function. For it we suggest the expression

$$\langle v(y, t)\, v(y', t-\tau) \rangle = Ry^2 y'^2 \exp(-\tau/\Delta), \tag{12}$$

where R and Δ are some constants. Equation (12) allows for the time damping of the velocity correlation and agrees with the known behavior of the amplitude of pulsations in the viscous sublayer. Such an approximation allows one to integrate the expressions for a_t and ν_t in elementary functions. As a result, for the ratio ν_t/a_t, i.e., for the turbulent Prandtl number, we obtain the expression

$$\mathrm{Pr}_t = \mathrm{Pr}\, \frac{1 + \frac{y_+^2}{2\Delta} - \exp(-y_+/\sqrt{\Delta})}{1 + \frac{\mathrm{Pr}\, y_+^2}{2\Delta} - \frac{\Lambda}{1+\Lambda}\exp(-y_+\sqrt{\mathrm{Pr}/\Delta})}. \tag{13}$$

The behavior of the quantity Pr_t for $\Lambda = 0$ and ∞ and different Pr is shown in Fig. 1a. The quantity Δ in Eq. (13) is dimensionless; we took $\Delta = 20$. This value follows from the experiments of [7].

We note an interesting fact, first pointed out by Kutateladze [8] and then confirmed in [3-5], to wit:

$$\mathrm{Pr}_t \sim y_+^{-1} \quad \text{as} \quad \mathrm{Pr} \to \infty. \tag{14}$$

This means that the behavior of the coefficient a_t in the viscous sublayer with $\mathrm{Pr} \gg 1$ differs considerably from the behavior of ν_t: $\nu_t \approx \text{const}\, \nu y_+^3$, but $a_t \approx \text{const}\, \nu y_+^4$ (in accordance with the well-known Landau — Levich equation [9-10]). Equation (14) corresponds to curve 5 in Fig. 1.

The separation of the curves as a function of the parameter Λ is shown in Fig. 1b, for two values of Pr. As follows from (13), at $\mathrm{Pr} \gg 1$ the influence of this parameter on Pr_t is limited to a narrow region of $y_+ \lesssim \sqrt{\Delta/\mathrm{Pr}}$.

In the "logarithmic" region ($y_+ \gg 1$) the turbulent Prandtl number ceases to depend on y_+ and, according to most of the experimental data, arrives at a value close to 0.87 [11]. Equation (13) can be corrected with the experimental asymptotic behavior by introducing a factor of 0.87 into it.

In Fig. 1c, the vertical segments show the scatter of the experimental data from [12] (Pr = 8); curve 1 is plotted from the corrected Eq. (13); curves 2 and 3 are taken from [2] (theory); 4 and 5 are obtained theoretically in [13], where the calculations are based on the equations for the second moments and on certain closure hypotheses; the influence of the heat conduction of the wall was not taken into account in [13].

A comparison of the present work with [2, 13] leads to the conclusion that the main results of these theories agree well. The quantitative disagreements in the values of Pr_t are evidently connected with a difference in the initial equations. The data of more detailed experiments in the region of $y_+ \leqslant 1$ are needed for further development and refinement of the theory.

NOTATION

Pr, Pr_t, molecular and turbulent Prandtl numbers; ν, kinematic viscosity; a, coefficient of thermal diffusivity of the liquid; b, coefficient of thermal diffusivity of the wall; ρ_1, c_{p_1}, λ_2, density, specific heat capacity, and thermal conductivity of the liquid; ρ_2, c_{p_2}, λ_2, the same for the wall; $\Lambda = \sqrt{(\rho c_p \lambda)_2/(\rho c_p \lambda)_1}$, parameter characterizing the influence of the wall; $\theta(y, t)$, temperature pulsations in the liquid; $\varphi(y, t)$, temperature pulsations in the wall; T(y), average temperature of the liquid; u, v, pulsation components of velocity transverse and perpendicular to the wall; U(y), average velocity; v_*, dynamic velocity; t, time; y, distance to the wall; $y_+ = v_* y/\nu$, dimensionless coordinate; Δ, constant characterizing the damping rate of the velocity correlations; ν_t, a_t, turbulent viscosity and turbulent thermal diffusivity; the index 1 refers to the liquid and 2 refers to the wall.

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